Solution of Lecture 8

1. Since $A=1$, $B=2$, $C=-3$, we have $B^2-4AC=16>0$, and this is a hyperbolic equation.

① characteristic eq: $\dfrac{dy}{dx}=\dfrac{2\pm4}{2}=3$ or -1, we have $x+y=C_1$ or $y-3x=C_2$.

② variable transformation: $\xi=x+y$, $\eta=x-\frac{1}{3}y$

③ $u_x=u_\xi+u_\eta$, $u_y=u_\xi-\frac{1}{3}u_\eta$, $u_{xx}=u_{\xi\xi}+2u_{\xi\eta}+u_{\eta\eta}$,

$u_{yy}=u_{\xi\xi}-\frac{2}{3}u_{\xi\eta}+\frac{1}{9}u_{\eta\eta}$, $u_{xy}=u_{\xi\xi}+\frac{2}{3}u_{\xi\eta}-\frac{1}{3}u_{\eta\eta}$.

and the canonical equation is $\frac{16}{3}u_{\xi\eta}=0$. Therefore, the general sol^n. is given by $u=\phi(x+y)+\psi(x-\frac{1}{3}y)$, where ϕ and ψ are arbitrary functions.

④ Applying initial conditions gives

$$\begin{cases}\phi(x)+\psi(x)=\sin x\\ \phi'(x)-\frac{1}{3}\psi'(x)=x\Rightarrow\phi(x)-\frac{1}{3}\psi(x)=\frac{1}{2}x^2+k\end{cases}$$

where k is a constant. Therefore, we have

$$\phi(x)=\frac{3}{4}\left(\frac{1}{3}\sin x+\frac{1}{2}x^2+k\right),\quad \psi(x)=\frac{3}{4}\left(\sin x-\frac{1}{2}x^2-k\right),$$

which gives

$$u=\phi(x+y)+\psi(x-3y)=\frac{1}{4}\sin(x+y)+\frac{3}{4}\sin\left(x-\frac{1}{3}y\right)+y\left(x+\frac{1}{3}y\right)$$

2. If $x>2t$, $u=\frac{1}{2}\left[(x+2t)^4+(x-2t)^4\right]$,
and if $x<2t$, $u=\frac{1}{2}\left[(x+2t)^4-(2t-x)^4\right]$.