李世豪　数学学院

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参考文献：中文 → 数学物理方法 → Ref.

Office hour：Every Tuesday，10:00 am - 11:30 am

"Ref: Linear PDEs for Scientists & Engineerings" by Tyn Myint-U & Lokenath Debnath

Course Arrangement: ① Introduction (DEs in mathematical physics);

② Method of characteristics and separation of variables for 1^{st} order PDEs;

③ 2^{nd} order linear PDEs (wave eq., heat eq. and Laplace eq.)

课程特色：① 更侧重于计算而非定理的证明，主要是对方法的阐述；

② 更偏向于直觉的说明，尽量在讲述过程中弄懂（看情况讨论定理证明）.

Lecture 1. Introduction

1. Why DEs? (为什么要研究微分方程？)

- They describe the nature of science.
- mathematical models: ① statistical models; ② eqs.

In fact, when we deal with mathematical problems, we "always" assume that they are as simple as possible ⇒ toy model ⇝ complex system (关于怪波的数学解释)

Our DEs discussed in this class are toy models, so techniques are the main task of this course. (Toy models include: traffic flow; wave eq.; heat eq.; Laplace eq.

Techniques include: characteristics; separation of variables; d'Alembert formula; Green's function and integral transform methods)

2. What are DEs?

$$f(\underbrace{x, y, \cdots}_{\text{independent variables}}, \underline{u}, u_x, u_y, \cdots, u_{xx}, u_{yy}, u_{xy}, \cdots) = 0$$

u → an unknown func. ⇒ sol^n. of DE.

① 关于DE的阶：最高次偏导的阶，e.g: $u_{xx} + u_{yy} = 0$ 是2阶PDE，

$u_t + u_{xxx} + 6uu_x = 0$ 是3阶PDE.

② 关于DE的线性性 $\begin{cases} \text{线性} \\ \text{非线性} \end{cases}$

- 如果方程关于u的导数项仅与独立变量相关，我们称其为线性PDE.

e.g: $yu_{xx} + xu_y = 1$

事实上，现实中的现象大多都是非线性的，线性的模型都是"toy model". 然而非线性PDE的数学结构是复杂的，我们仅考虑1阶的非线性PDE (esp. 拟线性PDE)，关于2阶方程，我们仅考虑线性的.

仅关于最高次导数是线性的.

3. Initial condition (I.C.) / Boundary Condition (B.C.) of PDEs

- Why do we consider I.C. / B.C. of PDEs?

The general solution space of PDE is of infinite dimension.

Example: Try to solve $u_x - u_y = 0$.

Sol^n: By using the variable transformation $\begin{cases} \xi = x+y \\ \eta = x-y \end{cases}$, one knows

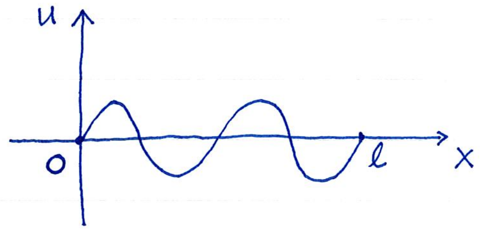
$$u_x - u_y = (u_\xi \xi_x + u_\eta \eta_x) - (u_\eta \eta_y + u_\xi \xi_y) = 2u_\eta = 0.$$

That is, $u = f(\xi)$ is a general solution of the above equation, where f is an ~~general~~ arbitrary function.

e.g: $u = x+y$, $u = \exp(x+y)$, $\cdots$ are solutions of the above equation.

How to fix this problem? - B.C or I.C.

e.g.: $\begin{cases} u_{tt} - u_{xx} = 0, \quad 0 < x < l, \ t > 0 \\ u(x,0) = \sin(\frac{2\pi x}{l}), 0 \le x \le l \ \text{(initial condition)} \\ u(0,t) = u(l,t) = 0, \quad t \ge 0 \ \text{(boundary condition)} \end{cases}$



H.W. 1: To verify that $u = f(x+ct) + g(x-ct)$ is a solution of $u_{tt} - c^2 u_{xx} = 0$, where f and g are arbitrary functions.

2. Find the solution of the initial-value problem

$u_{tt} - c^2 u_{xx} = 0$, $x \in \mathbb{R}$, $t > 0$, $u(x,0) = \sin x$, $u_t(x,0) = \cos x$.

(Answer: $u(x,t) = \sin x \cos ct + \frac{1}{c} \cos x \sin ct$.)

4. 关于PDE的解:

well-posedness: (existence, uniqueness, continuity)

How to solve PDEs: ① Exact solutions; ② Numerical solutions